Accurate Conductor Q-Factor of Dielectric Resonator Placed in an MIC Environment

R. K. Mongia, *Member, IEEE,* and P. Bhartia, *Fellow, IEEE*

***Abstract*— The unloaded Q-factor of a dielectric resonator is degraded due to conductor loss when it is placed in an MIC environment. In this paper, we report approximate but quite accurate closed form expressions for the Q-factor due to conductor loss. The results obtained are in good agreement with those of rigorous numerical methods. The effect of geometrical parameters on the useful tuning range of the structure is studied. Finally, an explicit relationship between the resonant frequency sensitivity factors and the conductor Q-factor is derived.**

I. Introduction

The intrinsic Q-factor of a dielectric resonator, which is determined by the dielectric loss of the resonator material, is usually very high. When a dielectric resonator is shielded by conducting walls as in all practical circuits, the unloaded Q-factor of the resonator is degraded due to the conductor loss. A dielectric resonator configuration which is widely used in practice is shown in Fig. 1. It is of considerable interest to designers to predict the effect of surrounding conductors on the unloaded Q-factor.

The unloaded Q-factor of the dielectric resonator placed in a shielded environment depends on the loss factor of the dielectric material and the conductor loss. With the continued progress in the development of low loss DR materials, the conductor loss becomes an important factor in determining the overall unloaded Q-factor. Rigorous numerical methods have been used in the recent past to compute the conductor Q-factor accurately [1]–[4]. However, these methods are quite complicated for a designer.

A perturbational method of computing the conductor Q-factor of the $TE_{01\delta}$ mode has been given by Kajfez [5]. This method determines the Q-factor indirectly by computing the relative change in resonant frequency when the conducting walls of the DR structure are moved inward by an amount equal to the skin depth. In principle, this is an excellent method if the resonant frequencies for the unperturbed and perturbed case are known accurately. In this scheme, one does not require to know the field distribution to compute the Q-factor. However, a problem may be faced in practice when using this method. Firstly, the perturbed and unperturbed frequencies are to be evaluated very accurately which may require use of a rigorous method. Moreover, even when a rigorous method is used for computing the resonant frequencies, the calculated values of resonant frequencies may vary from the true values because of inherent numerical errors present in any numerical method. Therefore, since the perturbed and unperturbed frequencies lie very close to each other, a lot of caution may be required when applying the Kajfez's method for computing the Q-factor.

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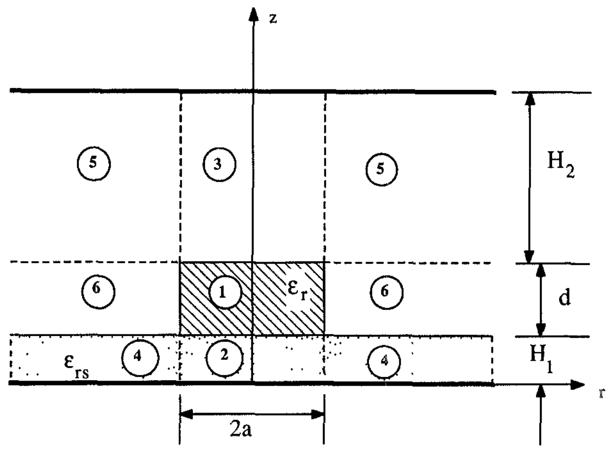

Fig. 1. Side view of a dielectric resonator placed in MIC configuration.

In this paper, we determine the conductor Q-factor by computing the stored energy and the power loss. The accuracy of Q-factor determined in this manner depends on the accuracy with which the field components are known. An effective dielectric constant method for analyzing the $TE_{01\delta}$ mode of structure as shown in Fig. 1 has been given in [6]. The method is an approximate one but is quite accurate. The fields calculated using this technique have been used to compute the conductor Q-factor. The results computed for conductor Q-factor are found to be in good agreement with those of rigorous methods. The effect of geometrical parameters on the useful tuning range of the resonator is also studied. Finally, an explicit relationship between the resonant frequency sensitivity factors and the conductor Q-factor is derived.

II. Method of Analysis

For the $TE_{01\delta}$ mode of the structure shown in Fig. 1, only three field components H_z, H_r, and E_ϕ exist. The H_z component from which the other field components can be derived can be written as follows for the field inside the resonator,

$$H_z = J_0(hr)[A\cos\{\beta(z-H_1)\} + B\sin\{\beta(z-H_1)\}] \tag{1a}$$

where $J_0(hr)$ is Bessel function of first kind and order zero. Further, h and β are the radial and z -directed wavenumbers inside the resonator. Their value is computed using the "effective dielectric constant" method as outlined in [6]. In other regions, the field component H_z can be written as

$$\begin{aligned} H_z &= J_0(hr)\sinh(\alpha z), \quad \text{in region 2} && (1b)\\ &= CJ_0(hr)\sinh\{-\gamma(z-H_1-d-H_2)\}, \\ &\qquad \text{in region 3} && (1c)\\ &= J_0(ha)\sinh(\alpha z)\,K_0(pr)/K_0(pa), \\ &\qquad \text{in region 4} && (1d)\\ &= CJ_0(ha)\sinh\{-\gamma(z-H_1-d-H_2)\} \\ &\quad \cdot K_0(pr)/K_0(pa), \\ &\qquad \text{in region 5} && (1e)\\ &= J_0(ha)[A\cos[\beta(z-H_1)\} \\ &\quad + B\sin\{\beta(z-H_1)\}]\,K_0(pr)/K_0(pa), \\ &\qquad \text{in region 6} && (1f) \end{aligned}$$

where $K_0(\cdot)$ denotes modified Bessel function of second kind and order zero.

The decay constants α, γ and p are related to wavenumbers inside the resonator by the following equations:

$$\alpha^2 = (\epsilon_r - \epsilon_{rs})k_o^2 - \beta^2 \quad (2a)$$
$$\gamma^2 = (\epsilon_r - 1)k_o^2 - \beta^2 \quad (2b)$$
$$p^2 = (\epsilon_{\text{eff}} - 1)k_o^2 - h^2 \quad (2c)$$

where ϵ_{eff} is the dielectric constant of an infinite cylindrical waveguide of radius 'a' and is computed using the method described in [6].

The constants A, B and C appearing in (1a)–(1f) are given by, [7]

$$A = \sinh(\alpha H_1) \quad (3a)$$
$$B = \alpha\cosh(\alpha H_1)/\beta \quad (3b)$$
$$C = \{\sinh(\alpha H_1)\cos(\beta d) + \alpha\cosh(\alpha H_1)\sin(\beta d)/\beta\}/\sinh(\gamma H_2) \quad (3c)$$

It may be noted that (1a)–(1f) do not represent fields exactly. For the structure shown in Fig. 1, it is not possible to express rigorous field distribution using closed form analytical formulas. The field expressions given above are therefore an approximation to the true field distribution.

Q-factor

The conductor Q-factor for a resonant structure is defined as

$$Q_c = \frac{2\omega_0 We}{P_c} \quad (4)$$

where W_e is the time average stored electric energy inside the dielectric resonator and the surrounding medium. P_c is the total loss in the imperfect top and bottom conducting plates.

Stored Energy

The electric energy stored inside the dielectric resonator is computed as

$$W_{e1} = \tfrac{1}{4}\epsilon_r\epsilon_0 \int |E_\phi|^2\,dv \quad (5a)$$

The electric field intensity E_ϕ inside the resonator can be derived from H_z given by (1a). The integration in (5a) is straight forward and we get the following expression for W_{e1}

$$W_{e1} = \frac{\pi}{2}\epsilon_r\epsilon_0 C_1^2 C_2^2 I_1 I_2 \quad (5b)$$

where

$$I_1 = \frac{a^2}{2}[J_1^2(ha) - J_0(ha)\,J_2(ha)], \quad \text{and} \quad (6a)$$
$$I_2 = \frac{d}{2}\left[1 - \frac{\cos(2u+\beta d)\sin(\beta d)}{\beta d}\right] \quad (6b)$$

Further, C_1, C_2 and u are given by

$$C_1 = \omega_o\mu_o/h \quad (6c)$$
$$C_2 = \left[\sinh^2(\alpha H_1) + \frac{\alpha^2}{\beta^2}\cosh^2(\alpha H_1)\right]^{1/2} \quad (6d)$$

and

$$u = \tan^{-1}[\beta\tanh(\alpha H_1)/\alpha] \quad (6e)$$

Similarly, closed form expressions can be derived for the stored energy outside the dielectric resonator in the air region and the dielectric substrate. We shall not give them here as they are also easily derived. It may be remarked that owing to the high dielectric constant of the resonator material, nearly all the electric energy is stored inside the dielectric resonator. Even if electric energy stored outside the resonator is neglected, a small difference will result.

Power Loss

The tangential magnetic field, and hence, the conduction current on the top and bottom metal plates can be found from (1b)–(1e). Once the current is known, the power loss can be calculated using the following formula

$$P_c = \tfrac{1}{2}R_m \int |J_c|^2\,ds \quad (7)$$

where R_m is the real part of the surface impedance of the conductor given by

$$R_m = \sqrt{(\pi f_o\mu_o/\sigma)}$$

Using (7), the power loss in the bottom ground plane is computed as

$$P_{c1} = \pi R_m \frac{\alpha^2}{h^2}(I_1 + I_3) \quad (8a)$$

where I_1 is given by (6a), and

$$I_3 = \frac{a^2}{2}\frac{J_1^2(ha)}{K_1^2(pa)}[K_1^2(pa) - K_0(pa)\,K_2(pa)] \quad (8b)$$

Similarly, the conductor loss in the upper plate is found as

$$P_{c2} = \pi C^2 R_m \frac{\gamma^2}{h^2}(I_1 + I_3) \tag{9}$$

The overall conductor Q-factor is thus given by

$$\frac{1}{Q_c} = \frac{1}{Q_{c1}} + \frac{1}{Q_{c2}} = \frac{P_{c1}}{2\omega_0 We} + \frac{P_{c2}}{2\omega_0 We} \tag{10}$$

where Q_{c1} is the Q-factor due to power loss in the lower ground plane alone and Q_{c2} is the Q-factor due to power loss in the upper ground plane alone.

Approximate Algebraic Expressions for Bessel Functions

The expressions appearing above contain Bessel functions. They are readily available on main frame computers. But since they are not so widely used as other mathematical functions, they are not available on mathematical library functions of a PC. Otherwise the expressions for the conductor Q-factor are very straightforward and can be easily programmed even on a hand held programmable calculator. To take advantage of the straightforward equations, Bessel functions appearing in the above equations were curve fitted using commercially available software programs. The approximate algebraic expressions are found as,

$$\begin{aligned} J_1^2(x) - J_0(x)\,J_2(x) &\approx -0.799 + 1.0839x \\ &\quad - 0.34781x^2 + 0.034033x^3, \\ &\quad 2.4 \le x \le 3.8 \\ J_1^2(x) &\approx -0.7557 + 1.5717x \\ &\quad - 0.67428x^2 + .082394x^3, \\ &\quad 2.4 \le x \le 3.8 \\ e^x K_0(x) &\approx 1.1097x^{-0.42087}, .2 \le x \le 5 \\ e^x K_1(x) &\approx 1.7563x^{-0.71026}, .2 \le x \le 5 \end{aligned} \tag{11}$$

The range of parameters in which the above functions are valid is adequate for all practical situations.

III. Results

Comparison with Other Methods

The algebraic expressions given in this paper were used to compute the conductor Q-factor. The resonant frequency, ϵ_{eff} and associated wavenumbers required to be used in these expressions were determined using the effective dielectric constant method as discussed in [6]. All these quantities are also determined using algebraic expressions and do not require Bessel functions. Computations were first made for those structural parameters for which results of Q_c derived using rigorous methods are available. The results are shown in Fig. 2 for a dielectric image resonator. The results for the rigorous method were read from those of Kobayashi *et al.* [2] who have also verified the accuracy of their results by comparing with experimentally measured results. It is seen that the results of the present theory are in good agreement with those of Kobayashi *et al.* [2]. To further validate the theory, results

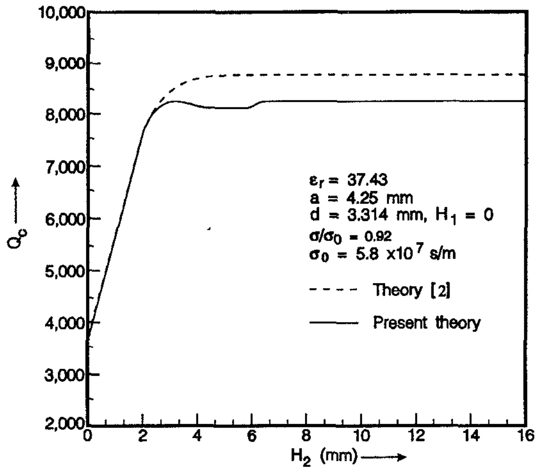

Fig. 2. Q-factor due to conductor loss of structure shown in Fig. 1. Comparison with results of Kobayashi, Aoki and Kabe [2].

were compared with those of Maj and Modelski who have employed rigorous mode matching method for the analysis [1]. Maj and Modelski do not give results explicitly for the conductor Q-factor. Instead they report results for the overall unloaded Q-factor. The overall unloaded Q-factor is given as

$$\frac{1}{Q_u} = \frac{1}{Q_d} + \frac{1}{Q_c} = \tan\delta_d + \frac{1}{Q_c} \tag{12}$$

The results reported in [1] are reproduced in Table I for several resonator and structural parameters. For the same parameters, the results were computed using the present closed form expressions and are given in the same table. It is seen that the agreement between the results is quite good.

Useful Tuning Range

The resonant frequency of the structure shown in Fig. 1 can be varied by changing the distance of the top metallic plate over the resonator. In practice, this is accomplished by a tuning screw which protrudes out of the top conducting plate. If the diameter of the tuning screw is much larger than that of the resonator, the tuning screw can be assumed to behave as a top conducting plate. By decreasing the distance of the tuning screw over the resonator, a large increase in resonant frequency can be achieved. However, this degrades the overall Q-factor of the resonator due to the increased conductor loss and also affects the temperature sensitivity of the resonant frequency. In this section, we discuss the effect of tuning on the conductor Q-factor. In the next section, we show that an explicit relationship exists between the conductor Q-factor and some of the resonant frequency sensitivity factors.

We define the useful tuning range as one in which the conductor Q-factor remains above a minimum value. Since the conductor loss can be controlled by adjusting structural parameters, the overall unloaded Q-factor of the DR structure is limited by the dielectric loss. It is desired that the conductor Q-factor be at least 5–6 times higher than the dielectric Q-factor so that the overall Q-factor is affected by a small

TABLE I
RESONANT FREQUENCY AND UNLOADED Q-FACTOR OF DIELECTRIC RESONATOR PLACED IN MIC ENVIRONMENT (FIG. 1) $\epsilon_{rs} = 9.6, H_1 = 0.7$ mm, $\sigma = 6.14 \times 10^7$ S/m

ϵ_r	$2a$ (mm)	d (mm)	H_2 (mm)	$\tan\delta_d \times 10^4$	f_o (GHz)		Q_u	
					[1]	Present Method	[1]	Present Method
34.19	14.98	7.48	0.72	3.02	4.348	4.34	2470	2473
34.21	13.99	6.95	1.25	3.19	4.523	4.50	2440	2457
34.02	11.99	5.98	2.215	3.47	5.050	5.01	2410	2423
36.13	6.03	4.21	10.10	4.22	8.220	8.14	1980	2181

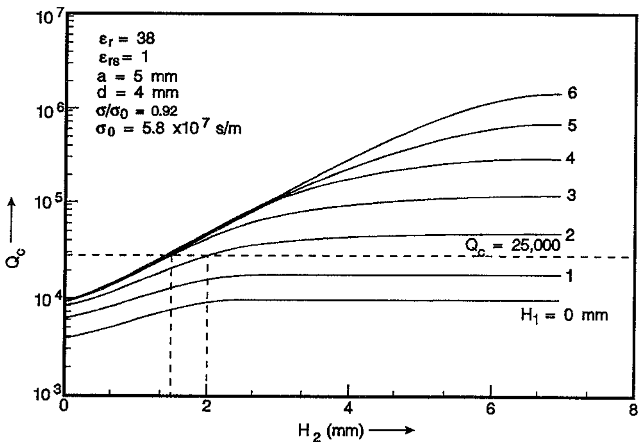

Fig. 3. Q-factor due to conductor loss of structure shown in Fig. 1.

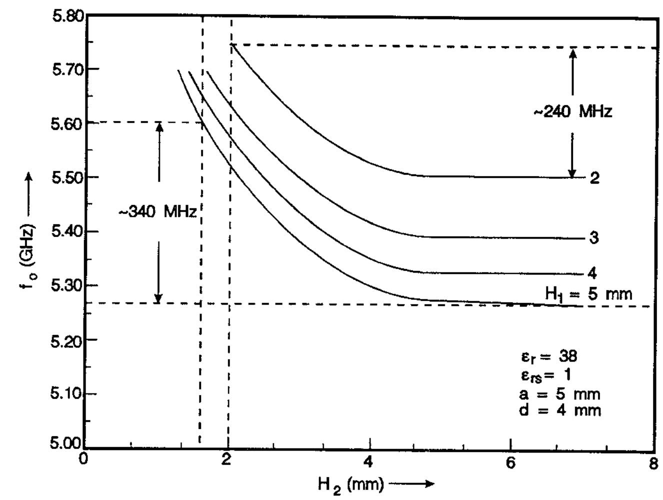

Fig. 4. Useful frequency tuning range of structure shown in Fig. 1.

amount only due to the conductor Q-factor. For a set of typical resonator parameters, we have shown some results for the conductor Q-factor and resonant frequency in Figs. 3 and 4 respectively. The results are shown as a function of parameter H_2 for various values of H_1. It is assumed that the dielectric resonator used has a value of Q_d of 5000 at about 5.5 GHz. We can then choose to have a minimum value of Q_c of about 25 000 which is about five times the value of Q_d. The useful tuning range can be found using Figs. 3 and 4. As an example, we shall first find useful tuning range for the case $\epsilon_r = 38, a = 5$ mm, $d = 4$ mm and $H_1 = 2$ mm. The distance H_2 is varied to achieve the frequency tuning. From Fig. 3, we see that the Q_c is higher than 25 000 for all values of $H_2 \geq 2$ mm. The value of the resonant frequency for $H_2 = 2$ mm is 5.74 GHz (Fig. 4). The resonant frequency approaches a limiting value of 5.50 GHz as H_2 is increased. Therefore, the useful tuning range for this case is about 4%. On the other hand, we see that if H_1 is chosen as equal to 5 mm, the useful tuning range is about 7%. Therefore, one can achieve a greater useful tuning range by judiciously choosing the value of H_1. In a practical circuit, H_1 can be easily controlled by keeping the dielectric resonator on a dielectric spacer as is frequently done to obtain a higher value of unloaded Q-factor.

IV. RELATION BETWEEN CONDUCTOR Q-FACTOR AND RESONANT FREQUENCY SENSITIVITY

The tuning of resonant frequency also causes a change in the sensitivity of the resonant frequency with respect to change in resonator and structural parameters. In this section, we derive an explicit relationship between the resonant frequency sensitivity factors and the conductor Q-factor. For the structure shown in Fig. 1, the sensitivity of the resonant frequency can be written as

$$\frac{\Delta f_o}{f_o} = S_a\frac{\Delta a}{a} + S_d\frac{\Delta d}{d} + S_{\epsilon r}\frac{\Delta\epsilon_r}{\epsilon_r} + S_{H1}\frac{\Delta H_1}{H_1} + S_{H2}\frac{\Delta H_2}{H_2} + S_{\epsilon rs}\frac{\Delta\epsilon_{rs}}{\epsilon_{rs}} \quad (13)$$

where S_x defines the sensitivity of resonant frequency with respect to parameter x. From the above equation S_{H1} can be written as

$$S_{H1} = \frac{\Delta f_o}{f_o}\frac{H_1}{\Delta H_1}\bigg|_{\Delta a=\Delta d=\Delta\epsilon_r=\Delta\epsilon_{rs}=\Delta H_2=0} \quad (14)$$

From [5], we know that if H_1 changes by one skin depth, $\Delta f_0/f_0$ is equal to $-1/Q_{c1}$. Therefore, if H_1 changes by $\Delta H_1, \Delta f_o/f_o$ is given by

$$\frac{\Delta f_o}{f_o} = -\frac{1}{Q_{c1}}\frac{\Delta H_1}{\delta_s} \quad (15)$$

where δ_s is the skin depth. From (14) and (15) S_{H1} is found as

$$S_{H1} = -\frac{1}{Q_{c1}}\frac{H_1}{\delta_s} \quad (16)$$

Similarly, we get for S_{H2}

$$S_{H2} = -\frac{1}{Q_{c2}}\frac{H_2}{\delta_s} \quad (17)$$

The above form of S_{H1} and S_{H2} is very elegant. Equation (17) shows that assuming that the tuning does not affect sensitivity factors other than S_{H2}, its effect on the sensitivity of resonant frequency would be negligible as long as $Q_{c2}\delta_s \gg H_2$.

V. Conclusions

In this paper, we have presented accurate closed form expressions for the Q-factor due to conductor loss of a dielectric resonator placed in MIC configuration. The effect of geometrical parameters on the useful tuning range is also studied. It is believed that expressions presented would be useful in practical applications.

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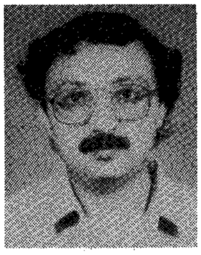

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